Name: RED ID:

Spring 2025 Math 245 Final Exam

Please read the following directions:

Please write legibly, with plenty of white space. Please **print** your name and REDID in the designated spaces above. Please fit your answers into the designated areas; material outside the designated areas (such as on this cover page) will not be graded. To get credit, you must also show adequate work to justify your answers. If unsure, show the work. The first four questions are worth 9-18 points, and the remaining sixteen questions are worth 10-20 points. The maximum possible score is $4 \times 18 + 16 \times 20$, for a total of 392 points. Since the exam is out of 390 points, it is possible to get as high as 100.5%. The use of notes, books, calculators, or other materials on this exam is strictly prohibited, except you may bring one 8.5"x11" page (both sides) with your handwritten notes. If you need scratch paper, you may use any blank space on your note sheet and on this front page. This exam will begin at 10:30 and will end at 12:30; pace yourself accordingly. Good luck!

Special exam instructions for SSW-1500:
1. Please stow all bags/backpacks/boards at the front of the room. All contraband, except phones, must be stowed in your bag. All smartwatches and phones must be silent, non-vibrating, and either in your pocket or stowed in your bag.
2. Please remain quiet to ensure a good test environment for others.
3. Please keep your exam on your desk; do not lift it up for a better look.
4. If you have a question or need to use the restroom, please come to the front. Bring your exam. I cannot come to you unless you are sitting by an aisle, sorry.
5. If you are done and want to submit your exam and leave, please wait until one of the six designated exit times, listed below. Please do **NOT** leave at any other time. If you are sure you are done, just sit and wait until the next exit time, with this cover sheet visible.

Designated exam exit times:
10:50 "See you next semester"
11:10 "I wish I had studied more"
11:30 "One extra hour of drinking – worth it"
11:50 "Maybe this will be good enough"
12:10 "There is nothing more in my brain, let me out of here"
12:30 "I need every second I can get"

Problem 1. Carefully define the following terms:

a. divides

b. partition

Problem 2. Carefully define the following terms:

a. irreflexive

b. modular equivalence

Problem 3. Carefully define the following terms:

a. product order

b. linear extension

Problem 4. Carefully define the following terms:

a. function

b. codomain

Problem 5. Let p, q be propositions. Use a truth table to prove that $p \uparrow (p \uparrow q) \equiv p \to (p \wedge q)$.

Problem 6. Let $x \in \mathbb{R}$. Prove that if $x^2 + x$ is irrational then $5x$ is irrational.

Problem 7. Simplify the following proposition, to where only basic propositions are negated. Do not try to prove or disprove, only simplify! $\neg\forall x \in \mathbb{Z}\ \exists y \in \mathbb{Q}\ \forall z \in \mathbb{N},\ x < y + 1 \leq z.$

Problem 8. Let $\mathbb{R}^+ = \{x \in \mathbb{R} : x > 0\}$. Prove that

$$\forall \varepsilon \in \mathbb{R}^+\ \exists \delta \in \mathbb{R}^+\ \forall x \in \mathbb{R},\ |x - 2| < \delta \to |3x - 6| < \varepsilon.$$

Problem 9. Let $x \in \mathbb{R}$. Prove that $\lfloor -x \rfloor = -\lceil x \rceil$.

Problem 10. Consider the factorial "function" $! : \mathbb{N}_0 \to \mathbb{N}$. Prove that it is left-total.
NOTE: This is part of proving it's well-defined, do not assume it's a function.

Problem 11. Let a_n, b_n, c_n be sequences with $a_n = O(b_n)$ and $b_n = O(c_n)$. Prove $a_n = O(c_n)$.
NOTE: This is Thm 7.9 in the book, do not use the theorem to prove itself!

Problem 12. Prove or disprove: For all sets R, S, T, we have $S \subseteq (R \cup S) \cup (T \Delta R)$.

Problem 13. Prove or disprove: For all sets S, T, we have $2^{S \cap T} \subseteq 2^S \cap 2^T$.

Problem 14. Let $S = \{a, b, c\}$, and let R be a trichotomous, symmetric, and transitive relation on S. Prove that $R = R_{full}$.

Problem 15. Compute which $x \in [0, 13)$ satisfies $4^{64} \equiv x \pmod{13}$. Fully justify each step.

Problem 16. Find an equivalence relation on $\mathbb{Z}$ with exactly three equivalence classes, two of which are finite. Justify your answer.

Problem 17. Let R be a partial order on set S. Let $T \subseteq S$. Suppose a, b are both maximal in T. Prove that $a = b \vee a \| b$.

Problem 18. Find a partial order on $S = \{1, 2, 3, 4\}$ of height 2 and width 2. Give your answer as a Hasse diagram, and justify your answer.

Problem 19. Consider the function $f : \mathbb{N} \to \mathbb{Z}$ given by $f(n) = \begin{cases} n/2 & n \text{ even} \\ -(n-1)/2 & n \text{ odd} \end{cases}$. Determine, with proof, whether or not f is injective.

Problem 20. Let $F_1 : S_1 \to S_2$ and $F_2 : S_2 \to S_3$ be surjections. Prove that $F_2 \circ F_1$ is also a surjection. NOTE: This is Thm 13.15(c). Don't use the theorem to prove itself!